Abstract

Algorithmic game theory is a research field that integrates game theory and algorithm design. The major target is to design good algorithms in strategic environments. In this project, we are going to study a well-established problem in algorithmic game theory called Facility Location Games. In the most classic setting, the government (service provider) selects locations to place its facilities on a street where some strategic agents (consumers) with private information live. Agents want to be as close as possible to the facility, but they may report wrong information to the service providers in order to get more benefits. The objective of the service provider is to collect the agents' information and use a deterministic or randomized mechanism to decide where to build the facility so that certain objective values are approximately optimized, and the agents will not gain benefits by reporting false information.

In recent years, many extensions to the original facility location games have been proposed and studied by researchers. Most of them assumed that agents had uniform preferences, which cannot model the complex scenario in real application. In this proposal, we plan to study more diversified preferences of the agents, and the utility or cost of an agent with preference might be determined by a single facility or by many facilities. Suppose that the government plans to open two schools (or kindergartens) in a district, where one school pays more attention to music education, and the other school mainly focuses on physical education. The parents who plan to send their children to school have their own perspectives on education and thus have different preferences. The parents make decisions (on which school to choose) based on their preferences and the locations of the two schools: if their favorite school is much farther away from home compared to the less preferred one, they would choose the less preferred one instead, which implies a tradeoff between the distance and the preference. This is an example of a single-facility-dominated facility location game, where the cost or utility of the parents depends on the single facility (the school) they choose. Consider another case that the government plans to build a hospital and a clinic in a district. If a patient needs to visit the doctor in the hospital twice a week, and visit the clinic once a week, then we can model the case as an instance of many-facility-dominated facility location games. According to the frequency they visit hospital and clinic, we could assume that the patient prefers 67% of the hospital and 33% of the clinic. The utility of this patient should be a weighted sum over the utilities received from each facility, which is determined by many facilities. This is known as fractional preference. We extend to the following scenario where the government has a budget of money to assign to the hospital and the clinic, which can be used to maintain or improve the facilities. An immediate question is how to assign the budget in order to better serve the patients.

In this project, we study facility location games with ordinal and cardinal preferences. For ordinal preference case, the agents are required to report a ranking over the facilities, and their costs are decided by the facility that provides the least cost; for cardinal preference case, the agents are required to report a fractional (or proportional) for each

facility and the government needs to decide on a budget allocation to these facilities. Under well-defined system objectives, we aim to design deterministic and randomized strategy-proof mechanisms with good approximation ratios and provide lower bounds. In addition, we study both cardinal and ordinal preferences when the government needs to both locate the facilities and allocate the budget. Our study of these new preference dimensions will enrich our understanding of possibilities that can arise with agents' preference matrix with respect to facilities. Therefore, the study could provide a more complete picture of facility location games with the existence of diverse preferences. We also define unified cost for agents with cardinal preferences and agents with ordinal preferences, so that it is possible to allow these two types of agents to co-exist, further elevating the problem scope to a higher level of generality.